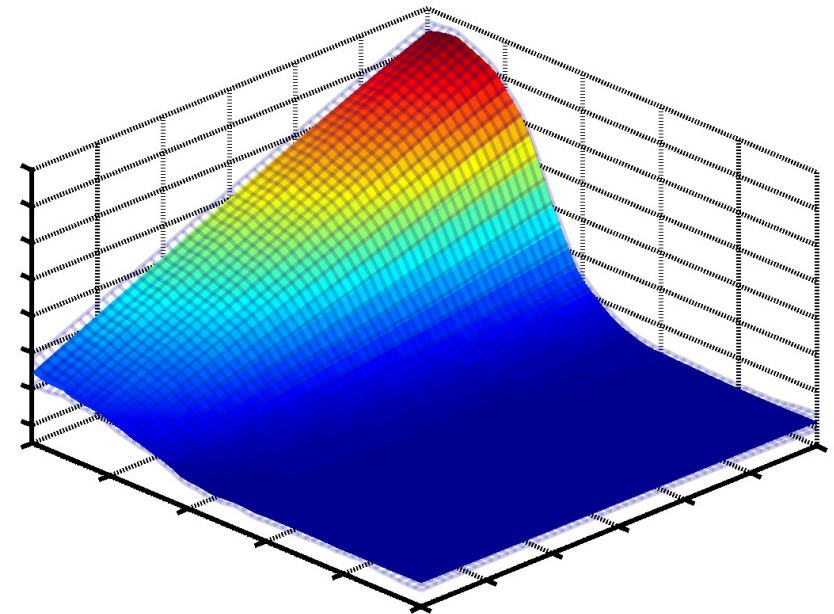
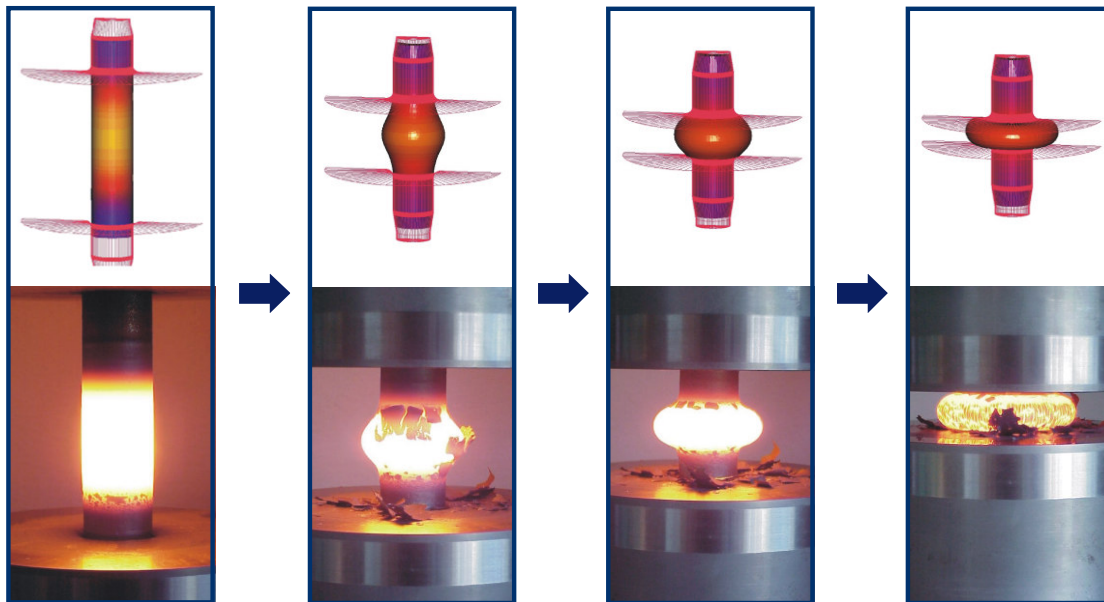


Analysis of a Thermomechanically Coupled Forming Process Using Enhanced Design and Analysis of Computer Experiments

Tobias Wagner, Christoph Bröcker, Nicolas Saba, Dirk Biermann, Anton Matzenmiller, and Kurt Steinhoff



A Comfortable Situation?



- A novel forming process
- Comprehensive real-world measurements
- A finite-element analysis providing high-resolution data
- A framework for Design and Analysis of Computer Experiments (DACE)

...But



The forming process

- features complex thermomechanically coupled effects

The experimental measurements

- are subject to noise
- cover only a part of the process

The finite-element analysis

- uses simplifications
- shows systematic deviations from reality

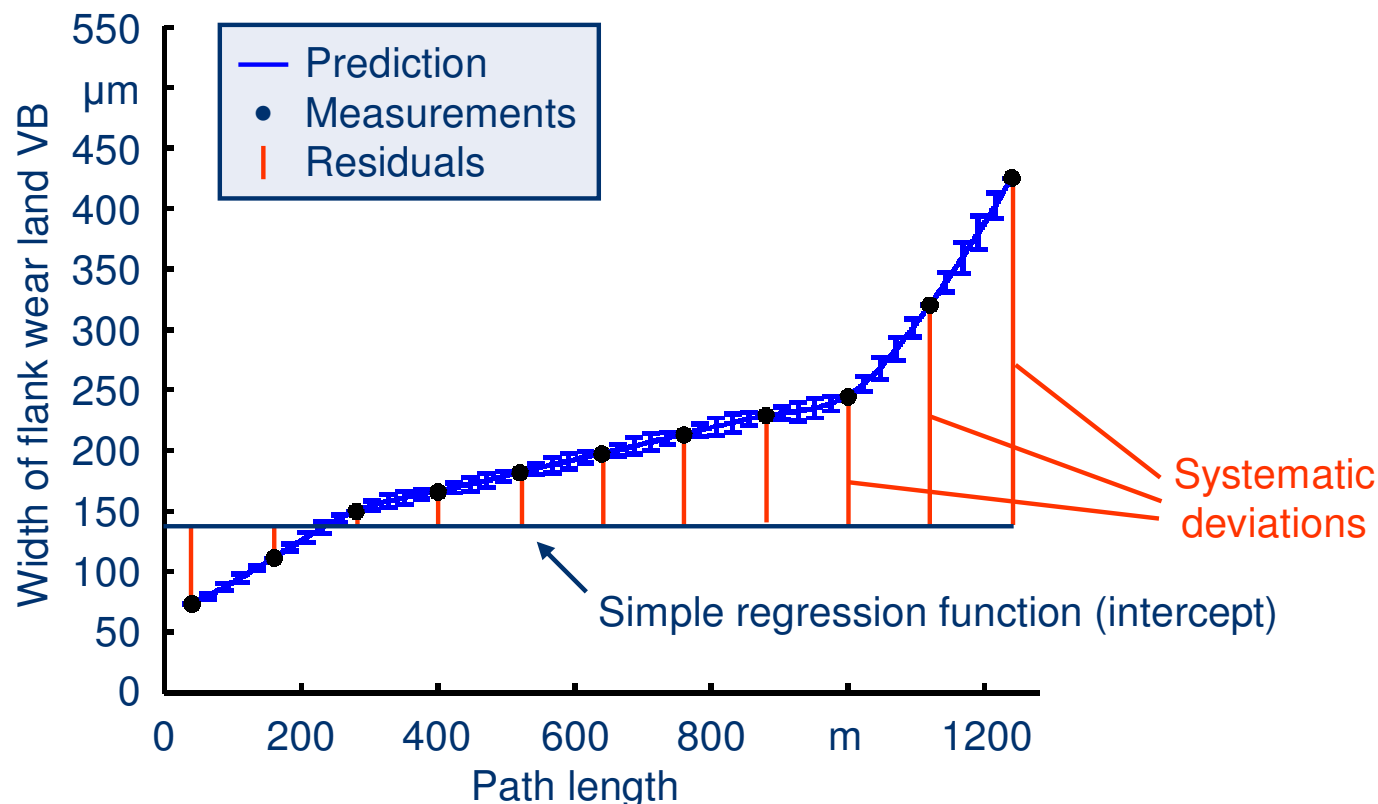
DACE

- is limited in the number of included observations

- Methodological Foundations
- Thermomechanically coupled forming process for functionally graded material
- Finite-element analysis of the forming process
- Combined DACE Metamodel
 - Empirical modelling of real-world measurements
 - Design and metamodeling of the finite-element simulation
 - Hierarchical two-level metamodeling

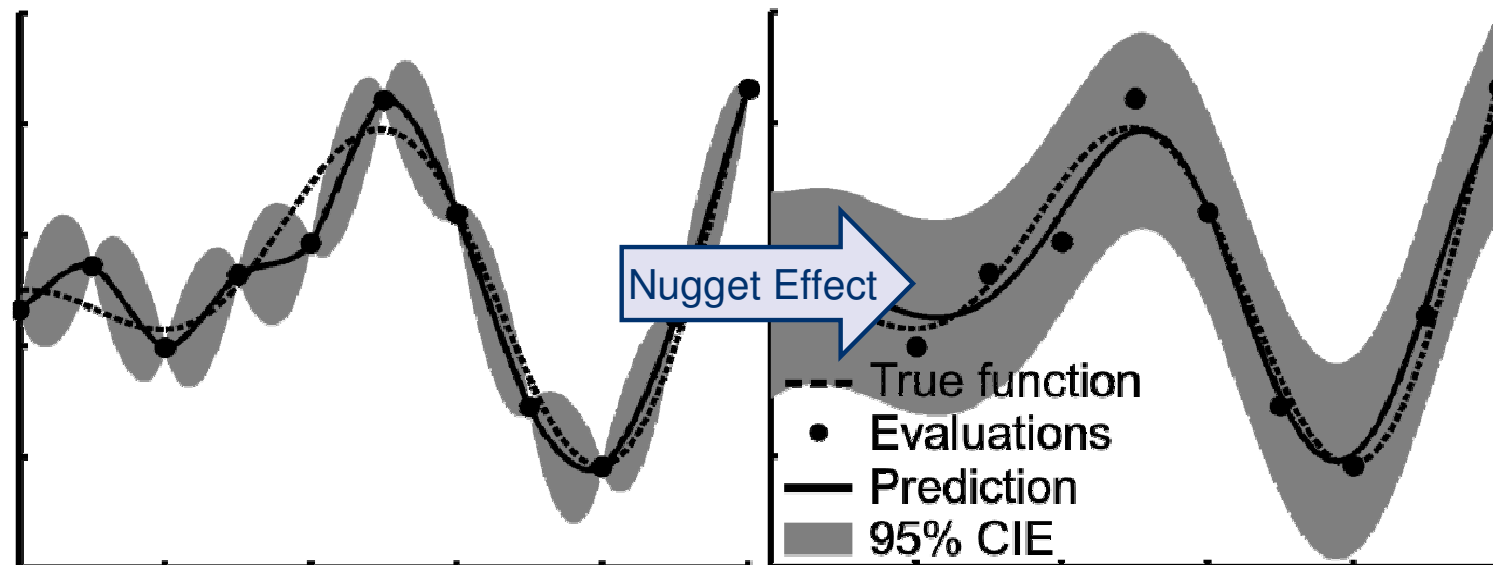
Methodological Foundations (DACE)

- Observations $y_i = \sum_{j=1}^p f_j(\mathbf{x}_i) \beta_j + Z(\mathbf{x}_i) = \mathbf{f}^T(\mathbf{x}_i) \boldsymbol{\beta} + Z(\mathbf{x}_i)$
- Predictions $\hat{y}_0 = \underbrace{\mathbf{f}_0^T \hat{\boldsymbol{\beta}}}_{\text{Regression function}} + \underbrace{\mathbf{r}_0^T \mathbf{R}^{-1} (\mathbf{y}^n - \mathbf{F} \hat{\boldsymbol{\beta}})}_{\text{Systematic deviations}}$

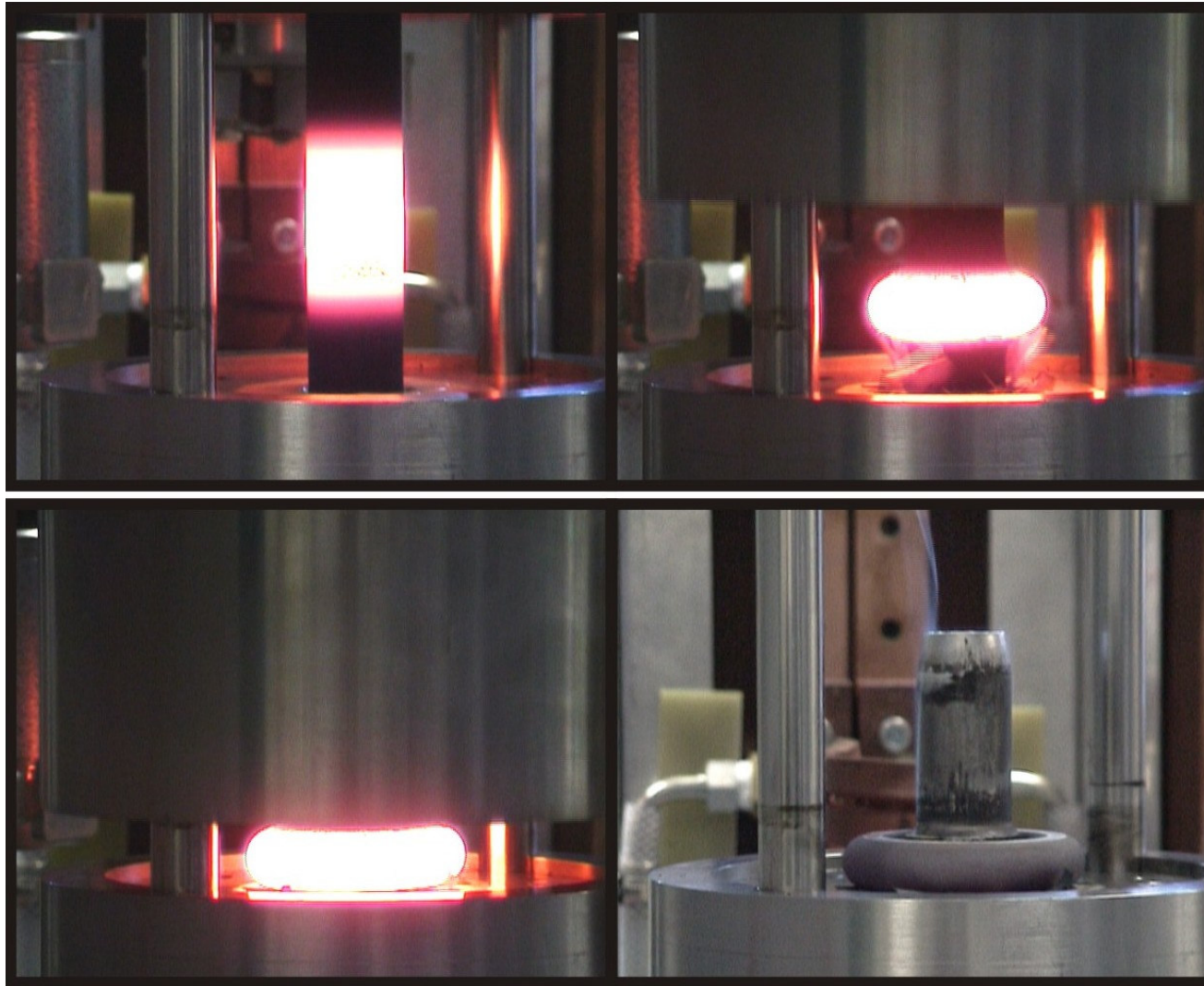


Methodological Foundations (Nugget Effect)

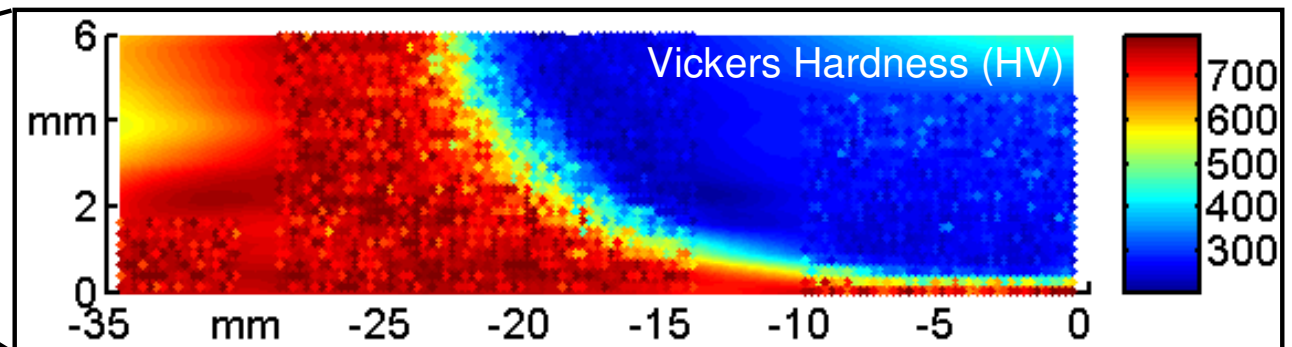
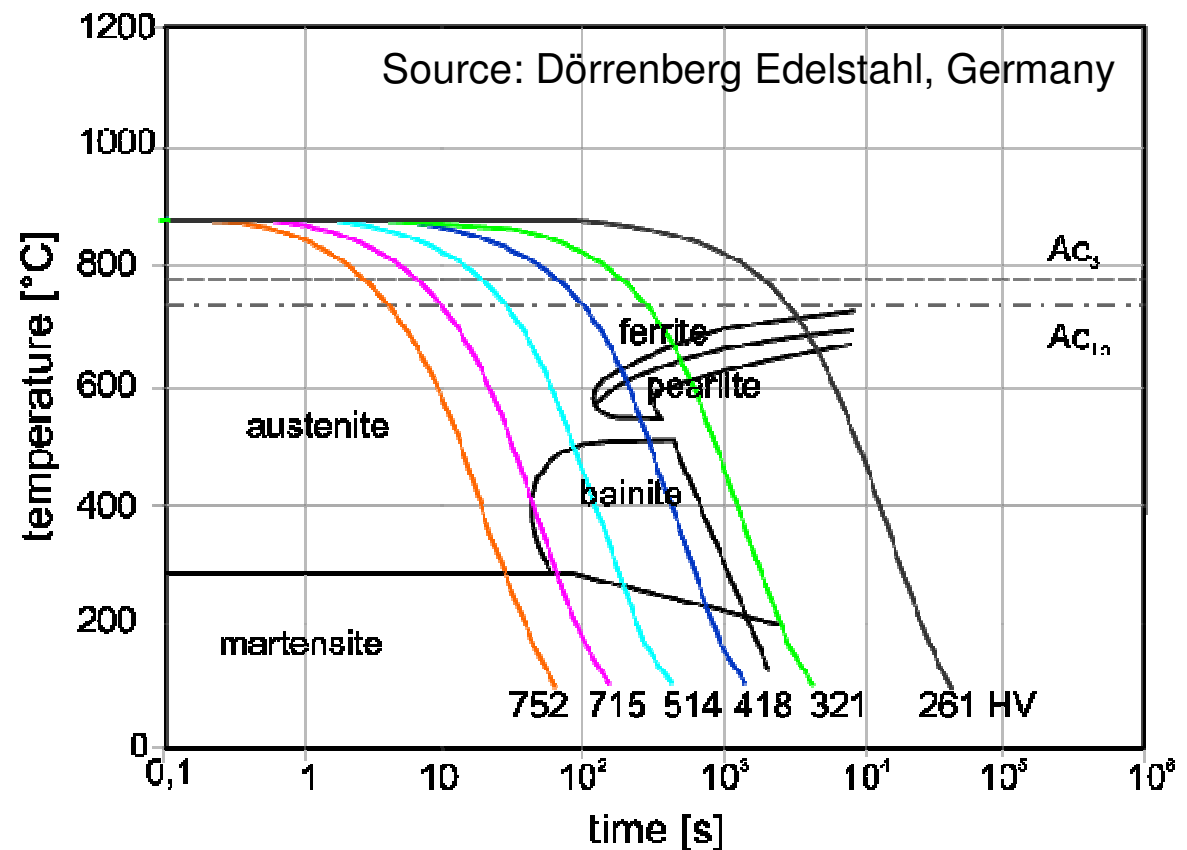
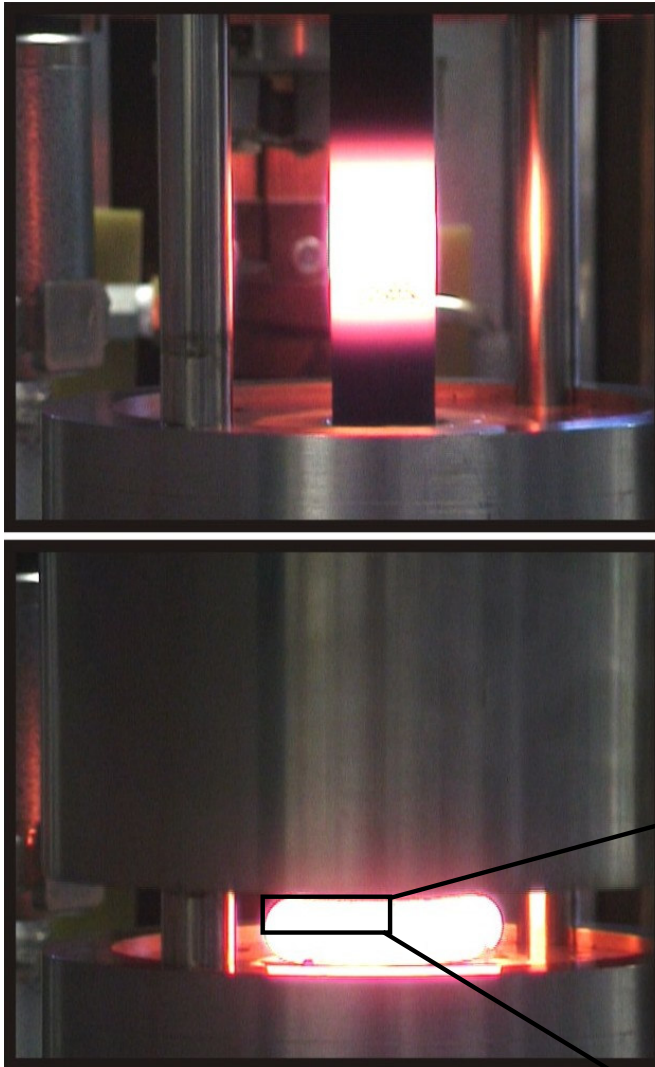
- Kriging correlation function $R(\mathbf{x}_i - \mathbf{x}_j) = \exp \left(- \sum_{k=1}^d \theta_k (\mathbf{x}_{i,k} - \mathbf{x}_{j,k})^{p_k} \right)$
- Nugget effect
$$\mathbf{R} = \begin{cases} 1 & i = j \\ gR(\mathbf{x}_i - \mathbf{x}_j) & i \neq j \end{cases}$$



Thermomechanically Coupled Forming

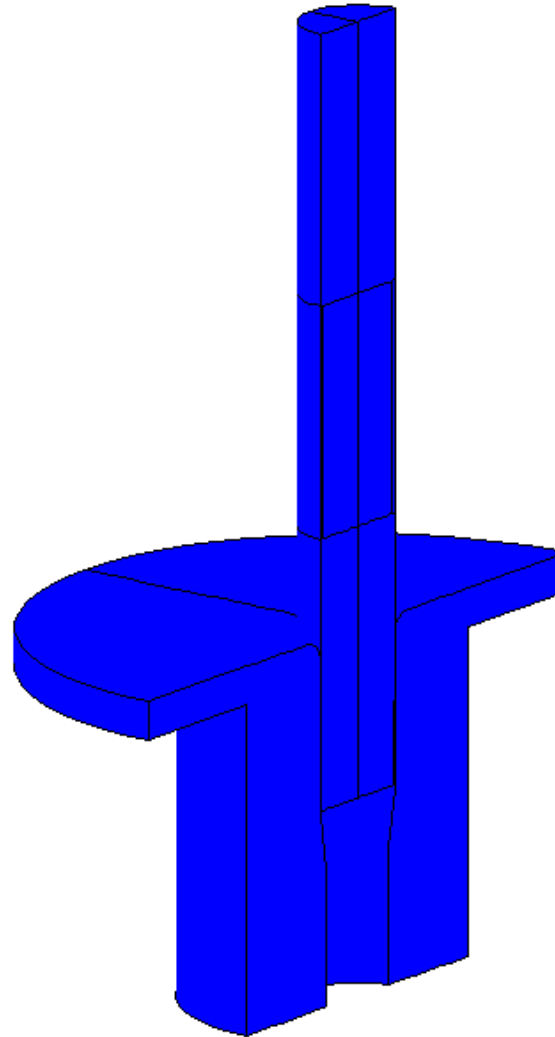


Functionally Graded Material

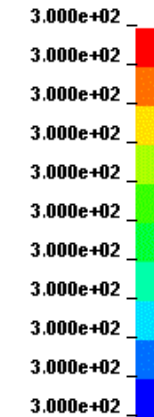


Finite-Element Analysis

FLANSCHWELLE
Time = 0
Contours of Temperature
min=300, at node# 155
max=300, at node# 155



Fringe Levels



- Methodological Foundations
- Thermomechanically coupled forming process for functionally graded material
- Finite-element analysis of the forming process
- **Combined DACE Metamodel**
 - Empirical modelling of real-world measurements
 - Design and metamodeling of the finite-element analysis (FEA)
 - Hierarchical two-level metamodeling

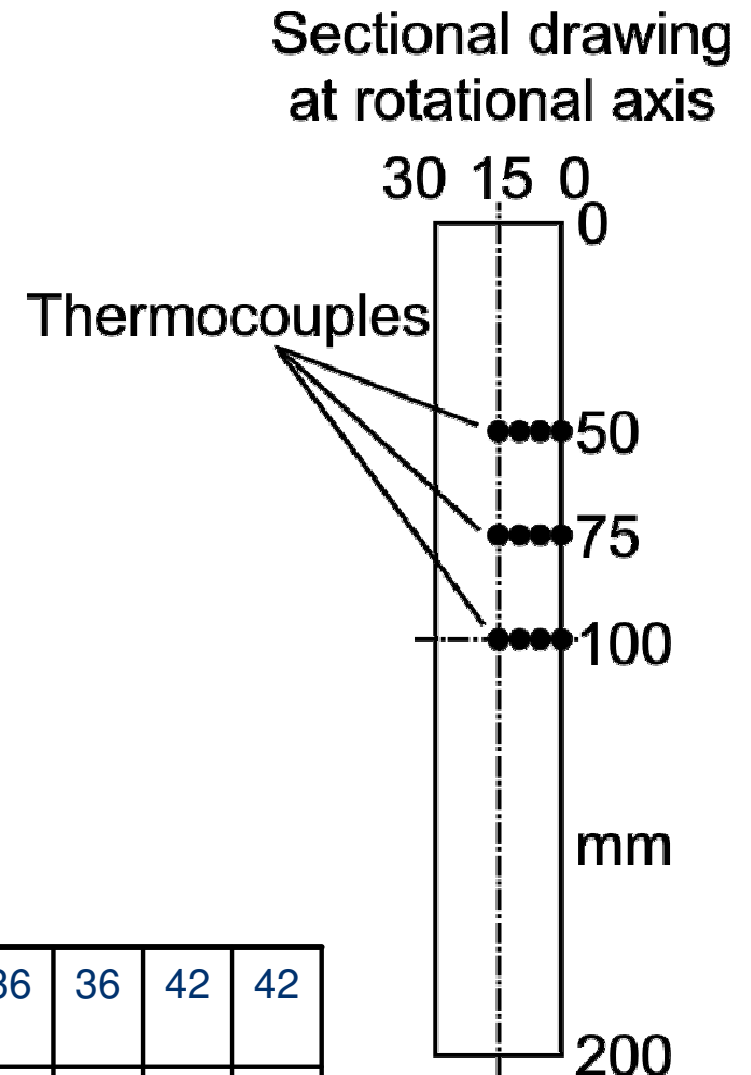
High deformation degrees

- Forming parameters fixed at maximum level
- Temperature distribution determines local cooling rates
- Temperature is measured by thermocouples
- No measurements during the forming process

Control variables (factors)

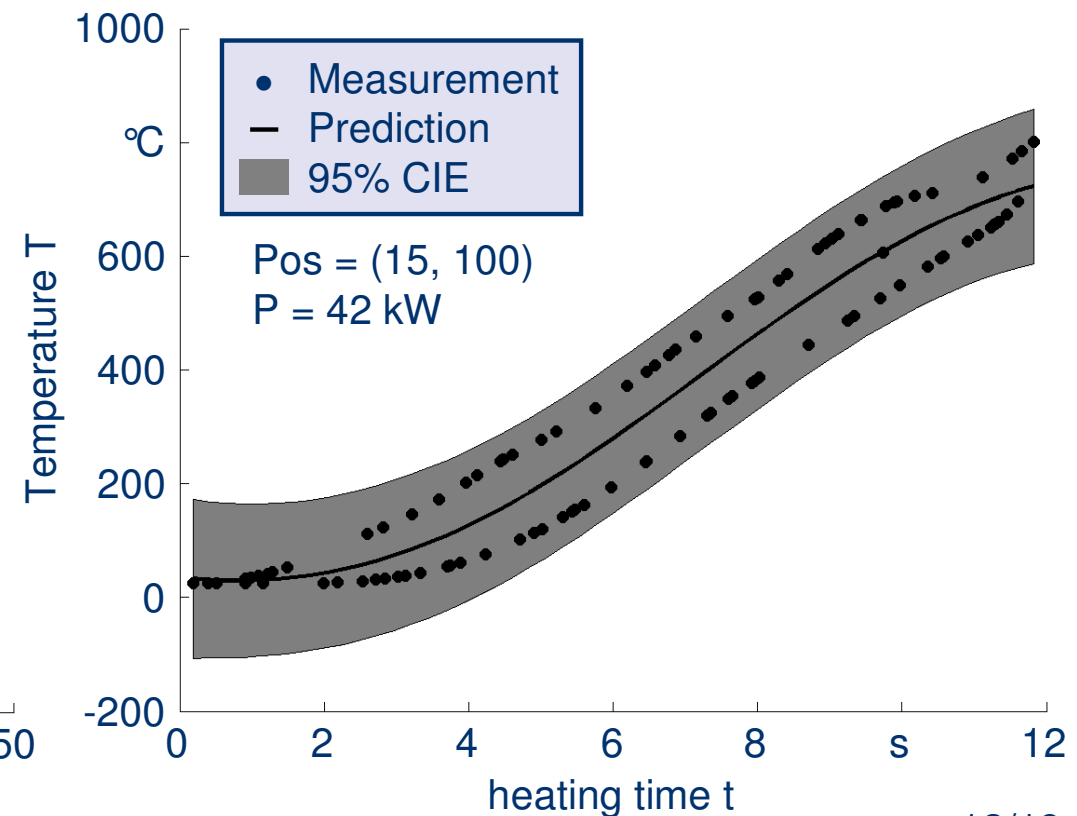
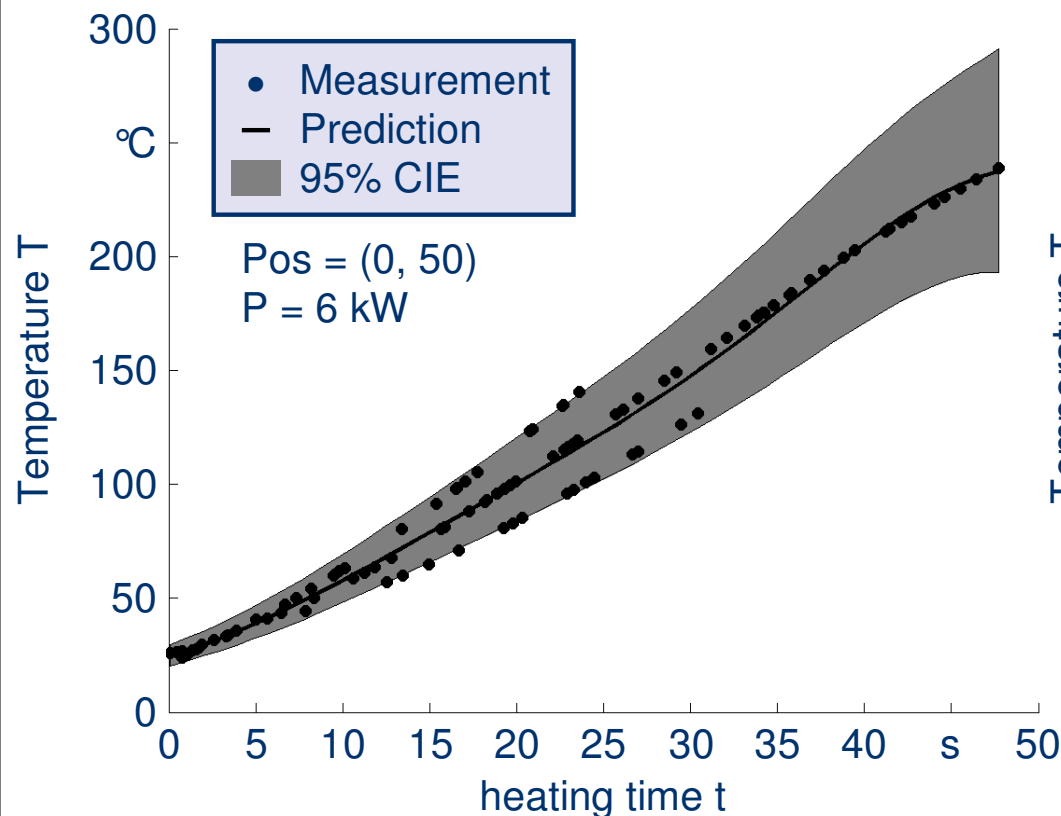
- Power of inductive heating P
- Heating time t

P [kW]	6	6	6	18	18	18	18	24	24	24	36	36	36	36	42	42
$t_{\max}$ [s]	24	32	48	6	12	24	32	6	12	18	3	6	9	12	12	12



Modelling of Real-World Measurements 2/2

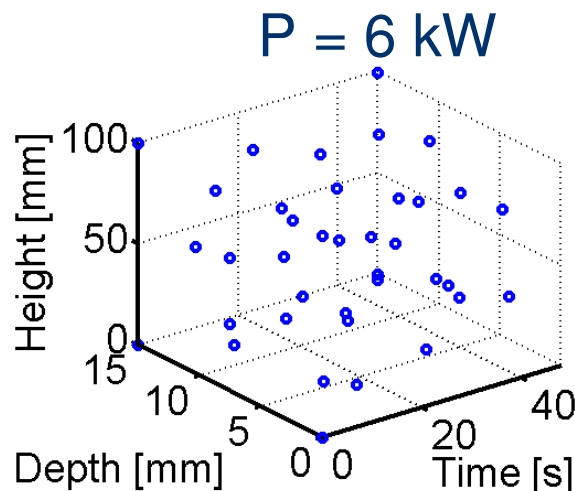
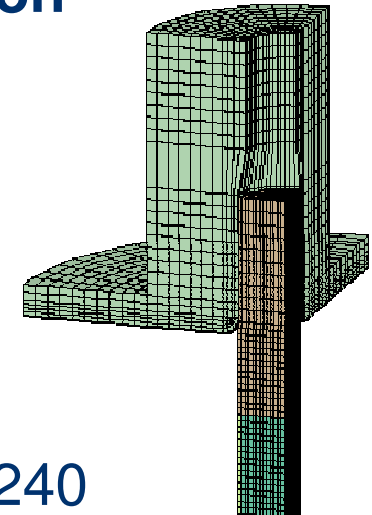
- Individual DACE models for each setting and measurement position
- Consideration of the nugget effect
- Random sampling of $n = 100$ observations without replacement
- Mean error over all empirical models of $11.8\text{ }^{\circ}\text{C}$



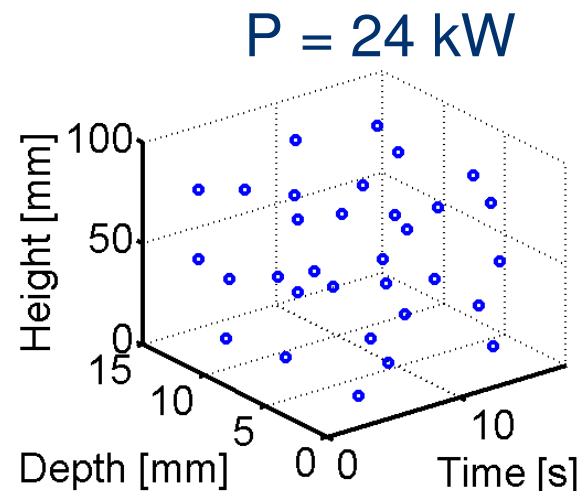
Metamodelling of the FEA Experiments 1/2

Global prediction of the resulting temperature distribution

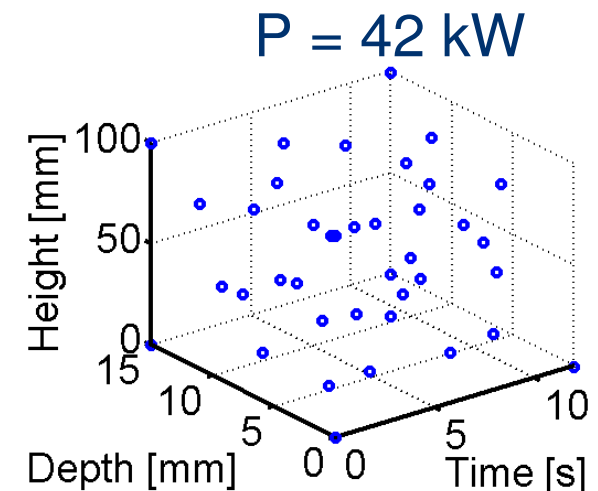
- Six runs of the FEA (for $P = 6 \text{ kW}$, 12 kW , ..., 42 kW)
→ 200,000 observations
- Optimized 3D Latin-Hypercube Designs (LHD)
for each level of P
 - Select data points closest to the 32 LHD samples
- Number of selected points $N = 7 \cdot 32 \text{ LHD} + 16 \text{ corners} = 240$



...



...



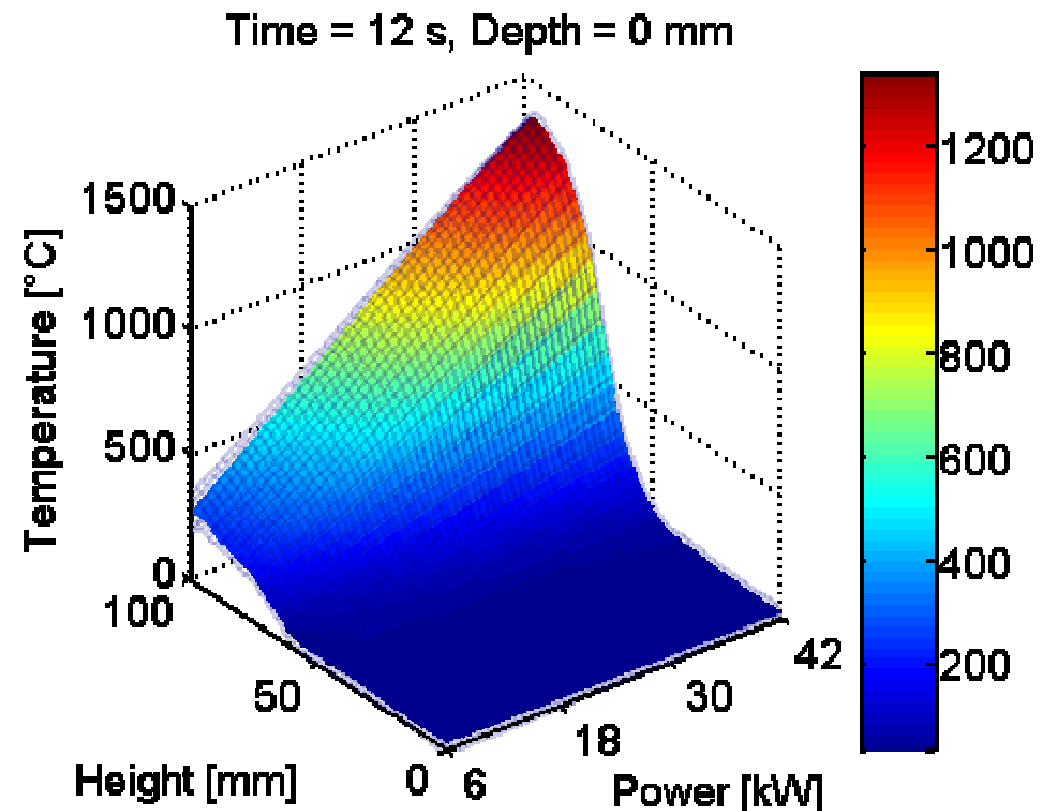
Modelling

- Fit of DACE metamodels (intercept) based on MLE
- No nugget effect is allowed since the output of the FEA is deterministic

Validation

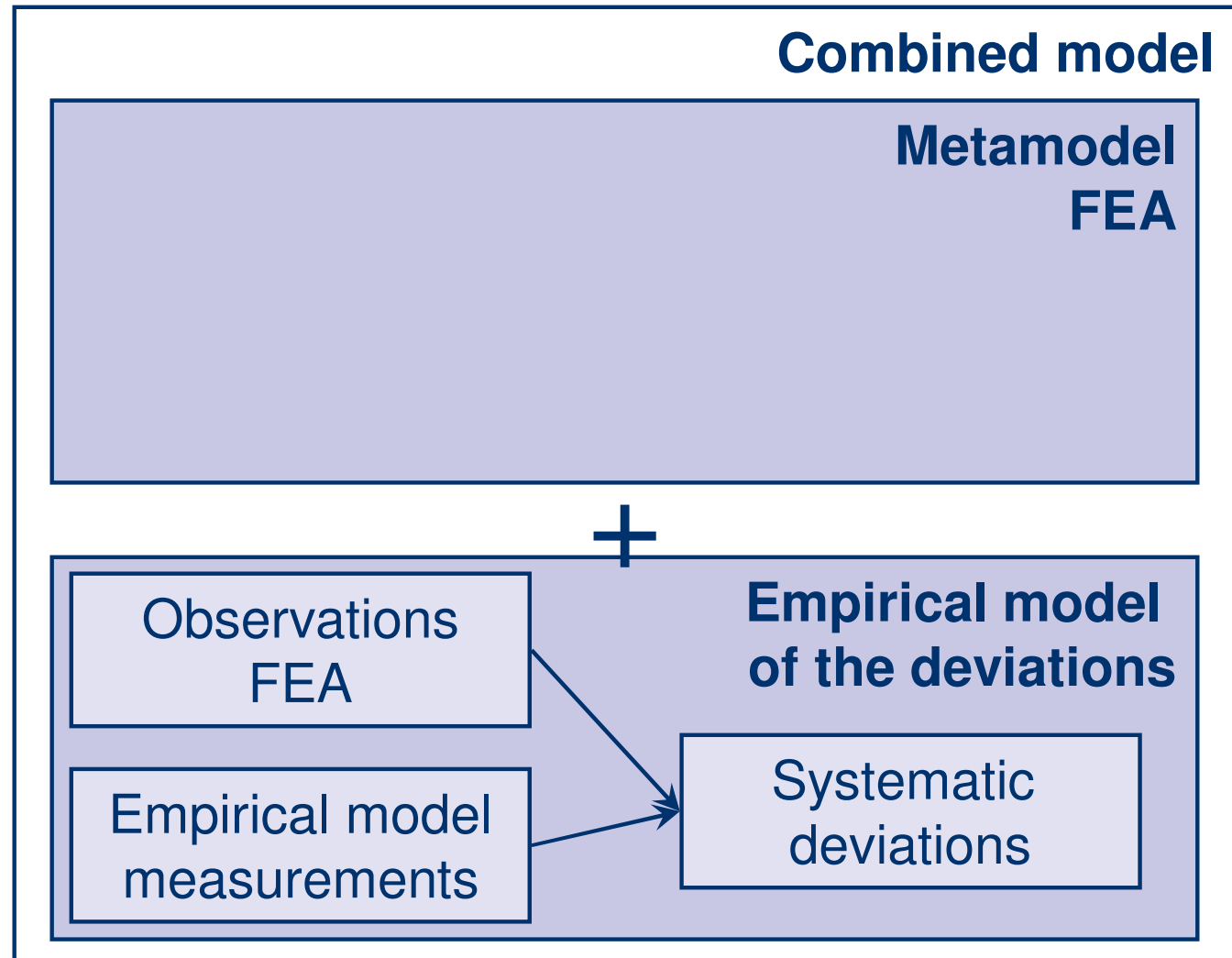
Mean error

- Gaussian correlation function
11.6 °C
- Kriging correlation function
7.2 °C



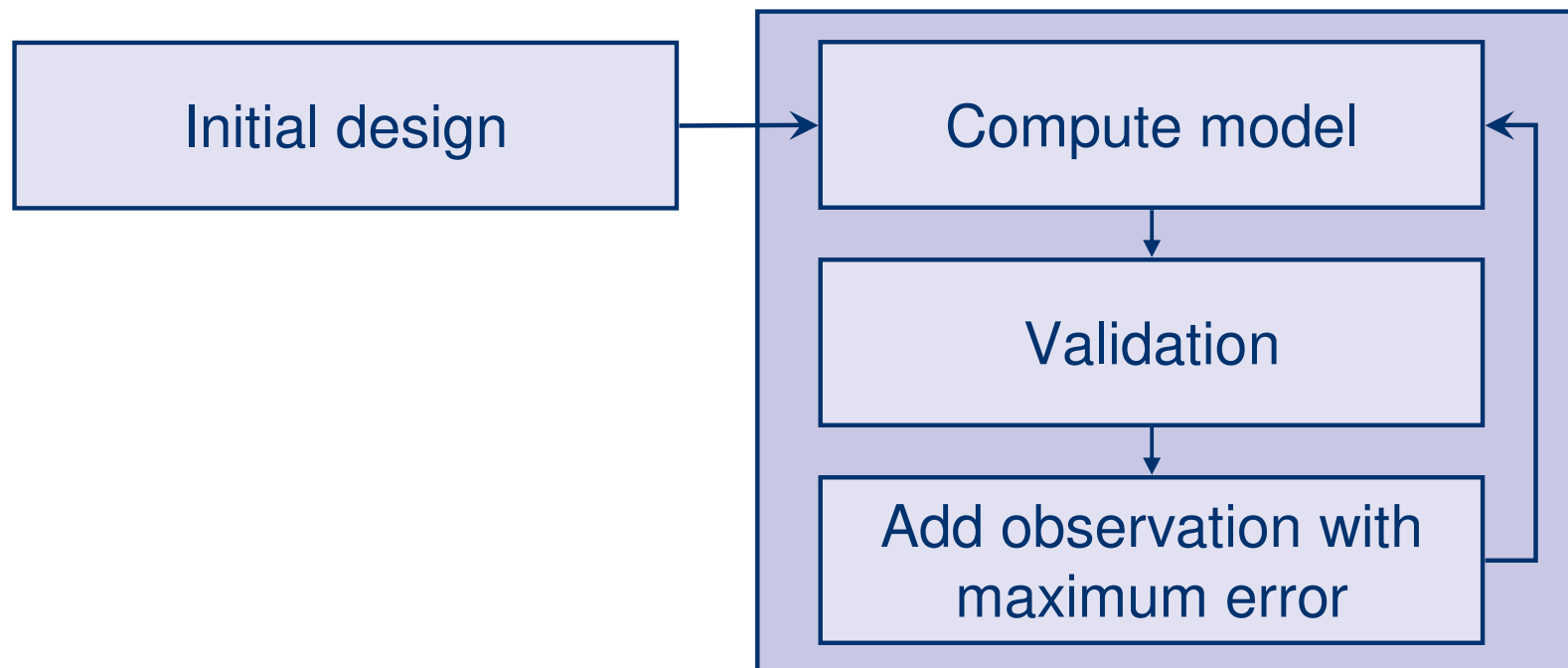
Procedure

- Compute deviations of the FEA based on the empirical DACE model for the measurements
- Use Metamodel of the FEA as regression function
- Add systematic deviations

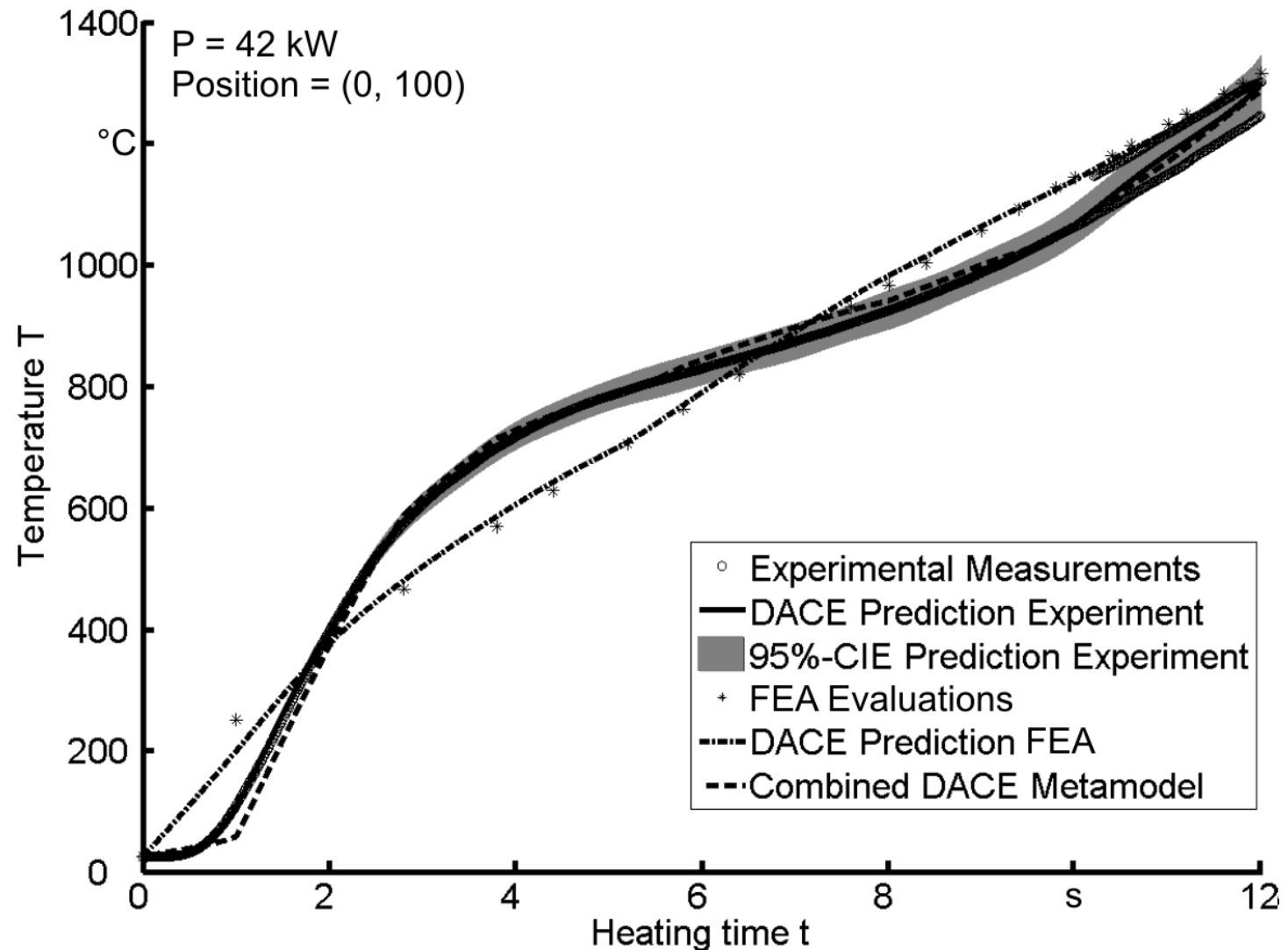


Sequential sample selection

- 2,964 observations for both measurements and FEA (1,354 are outside the 95% CIE of the empirical models)
- Complex nonlinear behaviour
- Positions are fixed by the experiments

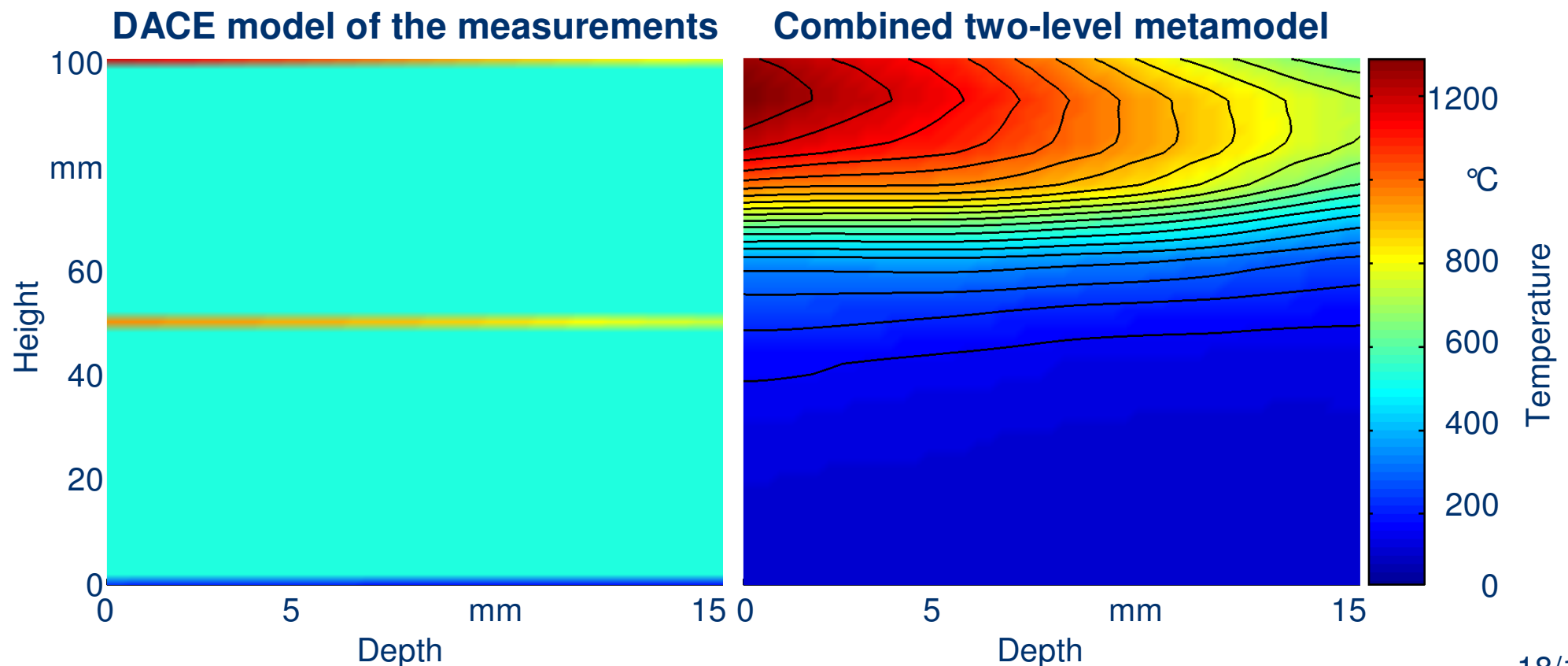


Validation



Comparison with a measurement-based model

- Regular and sparse distribution of measurement positions results in a singular Gaussian process model
- Combined model allows predicting the nonlinear temperature distribution



Summary

- Two-level metamodel of a complex inductive heating process based on
 - real-world measurements (no systematic modelling error)
 - output of a finite-element analysis (high resolution)
- Specific model enhancements
- Sample selection for each component model
- Validation

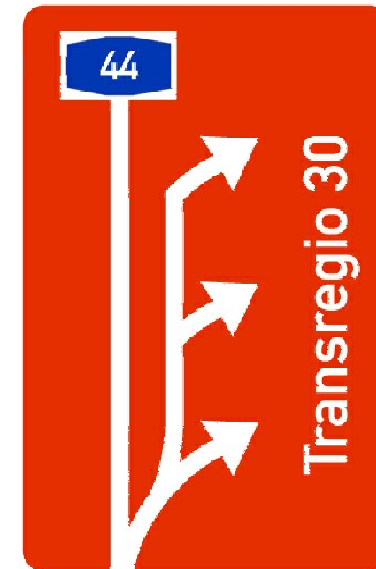
Outlook

- Improvement of the FEA model
- Violation of the assumption of normally distributed residuals
 - Nonstationary covariance models

Acknowledgement



Thank you for your attention



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